

Mathematics of finance

By Anurag Chauhan

finance: Art & science of management of money

Interest: cost of using others' money

waapas de to Denge na?



Mr. X
(Rich)
↓
Lender



Mr. Y
(Poor)
↓
Borrower

Mujhe udhaare Debo

RISK Taker
↓

Reducing His Liquidity
↓

Time value of money
↓

opportunity cost

wants
Return

wants
Reward

wants
compensation
for maintaining
same purchasing
power

↓
wants
at least
same
profit

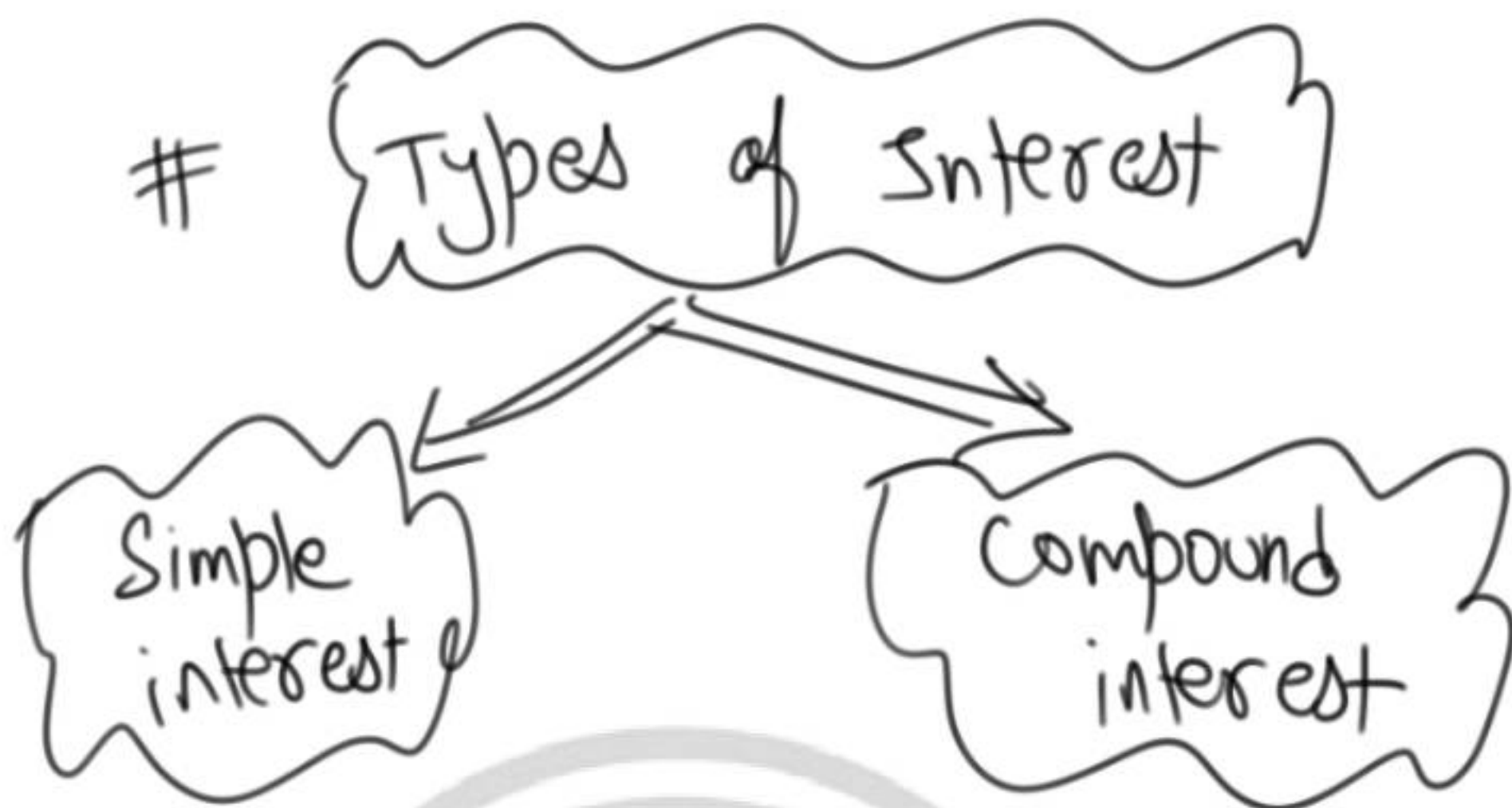
Because of all these
↓
"Interest is charged"

Some important terms

→ Principal = Borrowed money
(Invested money)

→ Amount or (Accumulated Amount)
= Principle + Interest

Amuray chankam sir



Simple Interest

No Interest on Interest

eg Mr. X Borrowed ₹10,000
@ 10% S.I. for 3 years.

Principle =	10,000
+ Interest for 1 st year	+ (1000)
<hr/>	
Amount after 1 st year	11000
+ Interest for 2 nd year	+ (1000)

Amount after IInd year 12000
+ Interest for IIIrd year + (1000)

Amount after IIIrd year 13000

Answer Chapter 18

Principle = ₹10,000

Interest for 3 years = 3000

Amount after 3 years = 13000

$S.I = P \cdot r \cdot t$

P = Principle

r = Rate of interest
in decimals

t = Time

Now Amount = P + I
= P + P r t

$$\# \text{ Amount} = P [1 + r t]$$

In Simple interest
for Double amount

$$r = \frac{1}{t} \quad \& \quad t = \frac{1}{r}$$

for Triple amount

$$r = \frac{2}{t} \quad \& \quad t = \frac{2}{r}$$

In Simple interest
when two time & two amounts
are given

$$\begin{array}{cc} t_1 & t_2 \\ A_1 & A_2 \end{array}$$

Then
$$r = \frac{A_2 - A_1}{A_1 t_2 - A_2 t_1}$$

In Simple interest
when two rates & two
amounts are given

r_1

r_2

A_1

A_2

Then

$j =$

$\frac{A_2 - A_1}{A_1 r_2 - A_2 r_1}$

Answer of Chauhan Sir

Compound Interest

Interest on Interest
is also calculated

Ms. X Borrowed £ 200,000
@ 20% for 3 years

Principle = £ 200,000

Int for 1st year = + 40,000

Amount after 1st year 2,40,000

Int. for 2nd year = + 48,000

Amount after 2 years 2,88,000

Int. for 3rd year + 57,600

Amount after 3 years 345,600

$$\left\{ \begin{array}{l} \text{Principle} = 200,000 \\ \text{Amount} = 3,45,600 \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Amount} = 3,45,600 \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{Interest} = A - P \\ \qquad \qquad = 1,45,600 \end{array} \right.$$

In compound interest

$$\text{Amount} = P \left[1 + \frac{r}{m} \right]^{t \times m}$$

$m = \text{No. of conversion period in a year}$

for Annually $\Rightarrow m = 1$

for Semiannually $\Rightarrow m = 2$

for Quarterly $\Rightarrow m = 4$

for monthly $\Rightarrow m = 12$

for Daily $\Rightarrow m = 365$

आगर Direct C.I. find
करना हो तो

$$\begin{aligned} \text{C.I.} &= A - P \\ &= P \left[1 + \frac{\alpha}{m} \right]^{t \times m} - P \end{aligned}$$

$$\text{C.I.} = P \left\{ \left(1 + \frac{\alpha}{m} \right)^{t \times m} - 1 \right\}$$

#

$$\text{C.I.} = P \left\{ \left(1 + \frac{\alpha}{m} \right)^{t \times m} - 1 \right\}$$

Calculator Tricks

#1) Square of Any Number

$$x^2$$

→ Press $\boxed{\times}$ शेक बार

→ Press $\boxed{=}$ शेक बार

#2) Cube of Any Number

$$x^3$$

→ Press $\boxed{\times}$ शेक बार

→ Press $\boxed{=}$ दो बार

#3) For $(x)^n = ?$

→ Press $\boxed{\times}$ one time

→ Press $\boxed{=}$ (n-1) times

Examples $(2)^{10}$

→ $\boxed{\times}$ one time

→ $\boxed{=}$ 9 times

#4] Reciprocal of Any Number

$(\frac{1}{n})$

→ $\boxed{\div}$ one time

→ $\boxed{=}$ one time

eg

$$\frac{1}{2} = 0.5$$

→ 2 on the screen

→ Press $\boxed{\div}$

→ Press $\boxed{=}$

#5] $(x)^{1/n} = ?$

→ Press $\sqrt{\quad}$ 12 times

→ Subtract 1 $\boxed{\text{Press } [-]}$

→ Divide by n

→ Add 1 $\boxed{\text{Press } [+]}$

→ Press \boxed{X} then $\boxed{=}$ → 12 times

eg $(8)^{1/3} \Rightarrow 8$ on the screen

→ Press $\sqrt{\quad}$ 12 times

→ -1

→ $\div 3$

→ +1

→ \sqrt{x}

x =
x =
x =
.
.
x =
12 times

g for (2) 4.5
→ 2 on the screen
→ Press √ √ √ √ ... 12 times
→ -
→ multiply by 4.5
→ +1
→ x =
x =
x =
x =
.
12 times

$$\#6] \log_{10}(x) = ?$$

- 5 19 times
- -1
- multiply by 227695

Remember Property of log

$$\log(x)^n = n \log x$$

$$\text{eg } \log(1.5)^{10} = 10 \log(1.5)$$

7] Antilog (x) = ?

→ ÷ 227695
→ +1
→ $\boxed{x=}$ 19 times

eg

$$P = 100$$

$$A = 200$$

$$r = 10\% \text{ Annually}$$

find time

Sol:

$$A = P \left[1 + \frac{r}{m} \right]^{t \times m}$$

$$200 = 100 \left[1 + \frac{0.10}{1} \right]^{t \times 1}$$

$$2 = (1.10)^t$$

Here power is variable

So we have to use log

By Anurag Chauhan srf

Put log Both side

$$\log 2 = \log (1.10)^t$$

$$\log 2 = t \log (1.10)$$

$$t = \frac{\log 2}{\log (1.10)}$$

now
use
log Trick

$$t = \frac{0.3010}{0.0414}$$

$$t = 7.27 \text{ years}$$

Qe

$$P = 100$$

$$A = 300$$

$$t = 8 \text{ years}$$

find compound rate of interest.

Sol:

$$A = P \left[1 + \frac{r}{3} \right]^{n \times m}$$

$$300 = 100 [1 + r]^8$$

$$(3)^1 = \underbrace{(1+r)^8}$$

Here base is variable, so transfer power 8 to left side

$$(3)^{\frac{1}{8}} = 1+r$$

$$(3)^{\frac{1}{8}} - 1 = r$$

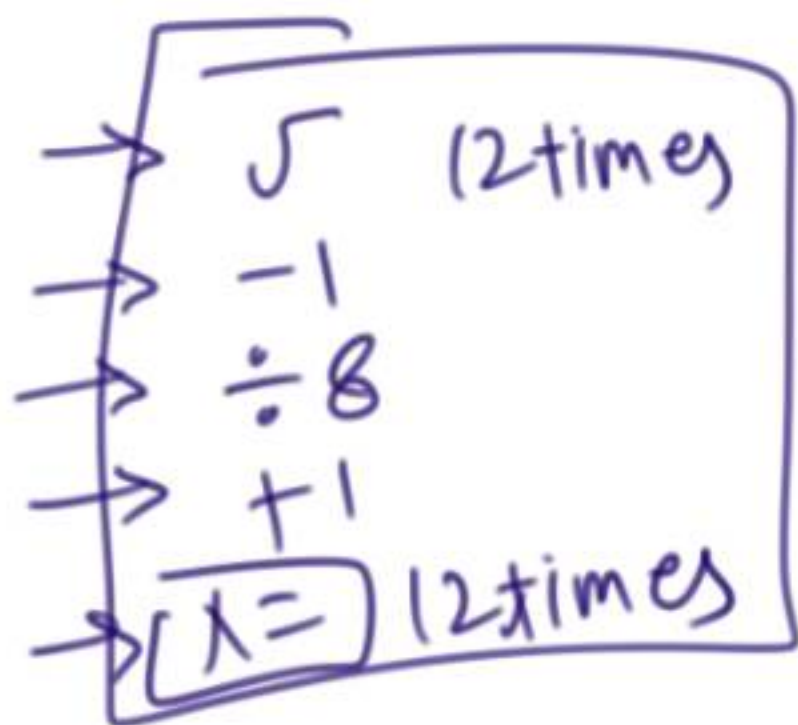
$$r = \underbrace{(3)^{\frac{1}{8}} - 1}$$

calculator trick

$$r = 1.1472 - 1$$

$$r = 0.1472$$

$$\text{or } 14.72\%$$



Depreciation



Decrease in value of Asset



$C =$ Cost

$d =$ Rate of Depreciation

$t =$ Time period for which asset is used

(Scrap value)

S.V. = value of Asset after ' t ' years

$$S.V. = C [1 - d]^t$$

Nominal Rate

Any Rate which is compounded

→ monthly

→ quarterly

→ semiannually

→ Daily

(Same interest kam dikhta
But interest Jaada milta hai)

Effective Rate

Any Annual compounding Rate

(Same interest jitna dikhta hai,
utna hi milta hai)

Nominal Rate can be converted to Effective Rate

$$r_e = \left[1 + \frac{r}{m} \right]^m - 1$$

Example

8% Semi annually is nominal rate

It can be converted in Annually

$$\begin{aligned} r_e &= \left[1 + \frac{r}{m} \right]^m - 1 \\ &= \left[1 + \frac{0.08}{2} \right]^2 - 1 \\ &= (1.04)^2 - 1 \end{aligned}$$

$$= 1.0816 - 1$$

$$= 0.0816$$

or
8.16%

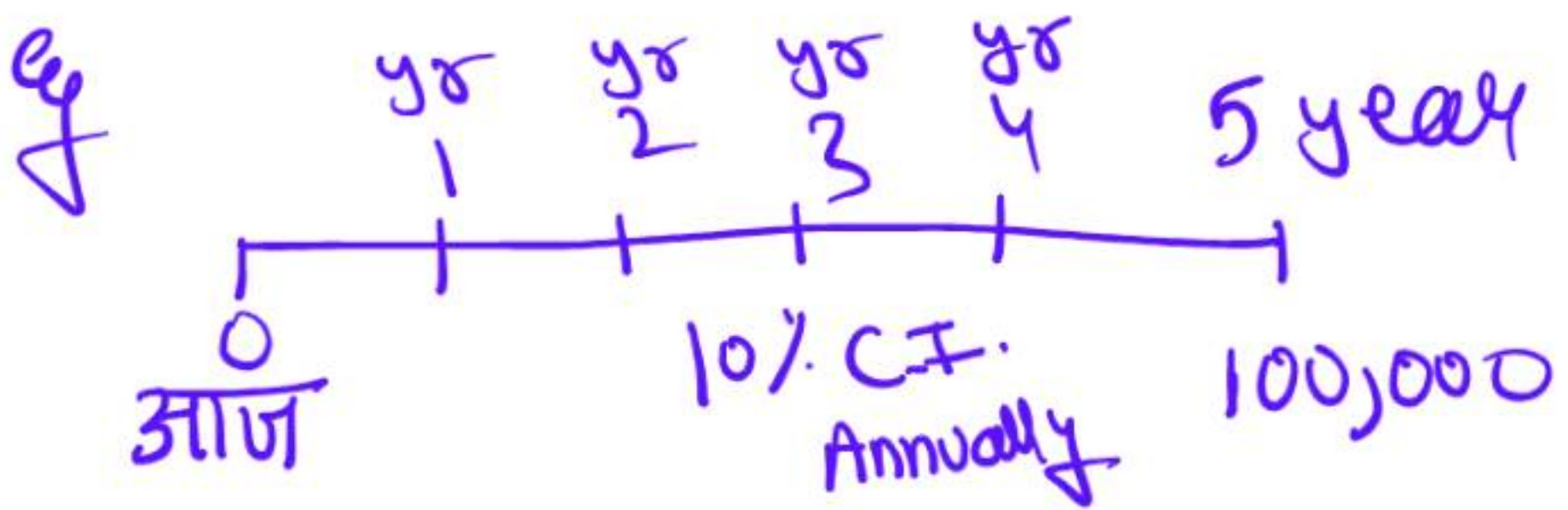
Effective
Rate

8% SemiAnnually \Rightarrow 8.16% Annually

Both are same

Present value of an Amount which is to be Received in future

$$\text{Present value} = \frac{\text{Future value}}{\left[1 + \frac{\delta}{m}\right]^n \times m}$$



5 years ke baad ₹ 100,000

midenge, lekin wo Paise agar

Agar midenge to ₹ 100,000
nahi milega, thoda kam milega

$$P.V. = \frac{100,000}{\left[1 + \frac{0.10}{1}\right]^5}$$

$$= \frac{100,000}{1.61051}$$

$$P.V. = 62,092$$

आज Bus इतना

ही मिलेगा

Annuity

→ Sequence of payments
(or receipts)

→ Same payment
(5000, 5000, 5000, 5000)

→ Same time interval
b/w two payments
(3m, 3m, 3m, 3m, ...)

g → LIC premium

→ House Loan EMI

Two Type of Annuities

Regular Annuity
(Ordinary)

when regular payment
are made at
the end of period

g House Loan EMI
(Car machine ke
end mein paise
Bank se katenge)

Immediate
(Annuity Due)

when regular
payments are
made in beginning
of each period

g LIC Premium
har year ki
starting mein
Bank se katenge

Regular Annuity

⇒ Future value : when benefit of many periodic investments is received in future

$$F.V_o = R \left\{ \frac{(1+i)^n - 1}{i} \right\}$$

R = Regular periodic payment

$$i = \frac{\sigma}{m}$$

$n = t \times m$ (Total payments)

Q You start investing £ 5000 every year in a Recurring Deposit which gives 10% C.I. . How much will you receive after 15 year?

Sol: Here you are investing 5000 every year (Assuming in the end)

↓
Benefit will be received after 15 years in future

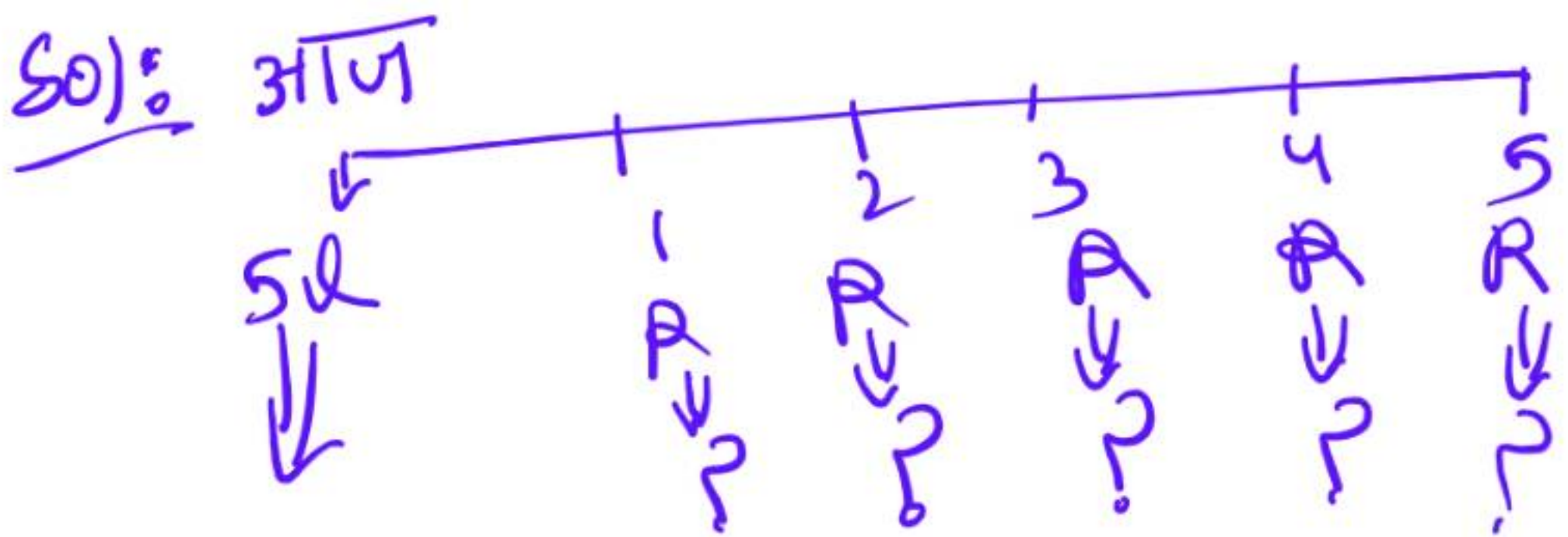
$$\text{Future value} = 5000 \left[\frac{(1+0.10)^{15} - 1}{0.10} \right]$$

$$= \text{£} 158,862$$

⇒ Present value : When some monetary benefit is received today then it repaid in installments.

$$P.V. = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

eg Ms. X took a loan of Rs 500,000 which is to be repaid in 5 yearly installments. Its rate of interest is 6% compounded Annually. Find the value of each installment.



माज Benefit

le Ma

↓
P.V.

$$500000 = R \left\{ \frac{1 - (1 + 0.06)^{-5}}{0.06} \right\}$$

$$500000 = R (4.2123)$$

$$R = \frac{500000}{4.2123}$$

$$R = ₹ 1,18,700$$

Calculator

$$(x)^{-n} \Rightarrow \text{Press } \left[\frac{x}{y} \right]$$

Press $\left[= \right]$ 'n' times

$$g \quad (1.06)^{-5} = 0.7472$$

Press $\left[\frac{x}{y} \right]$

Press $\left[= \right]$ 5 times

Immediate Annuity

$$F.V. = R \left[\frac{(1+i)^n - 1}{i} \right] (1+i)$$

$$P.V. = R \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)$$

Perpetuity

⇓
Infinite period

Annuity

Ordinary Perpetuity

$$P.V. = \frac{R}{i}$$

Immediate Perpetuity

$$P.V. = \frac{R}{i} (1+i)$$

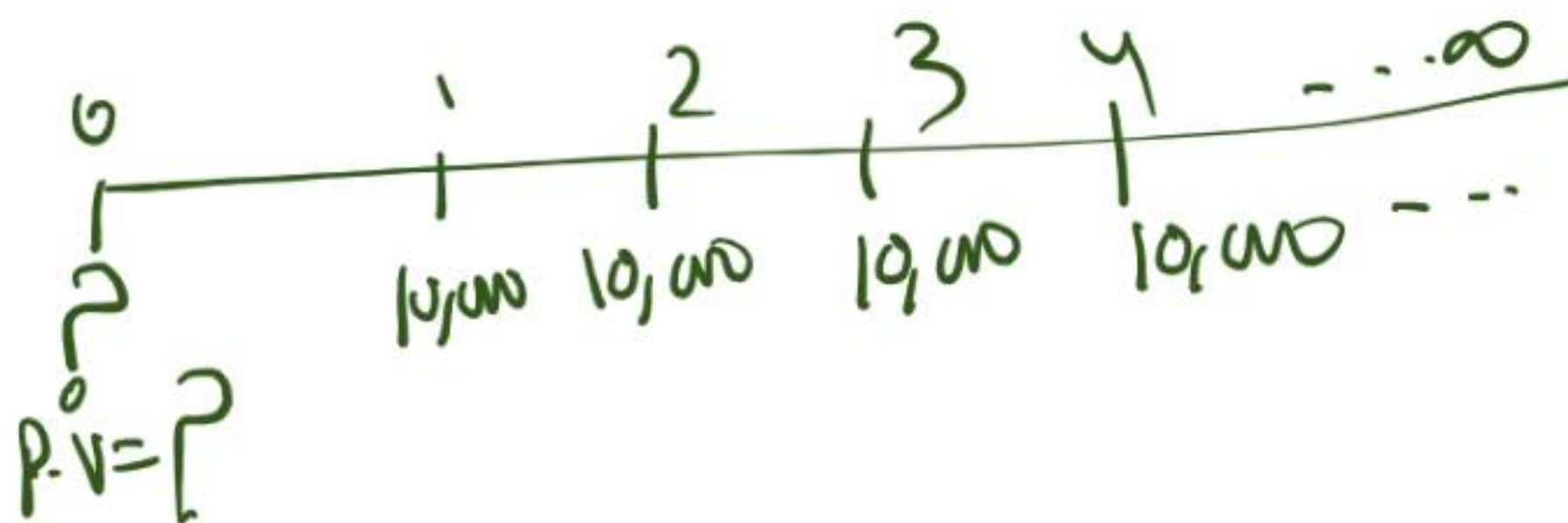
or

$$\frac{R}{i} + R$$

Q

How much you should invest now to receive £10,000 at the end of every year for indefinite period if rate of discount is 8% C.F. Annually?

Sol:



$$P.V. = \frac{A}{i} = \frac{10,000}{0.08} = 1,25,000$$

Q How much you should invest now to receive ₹ 10,000 in the beginning of every year starting from today for indefinite period if rate of discount is 8% C.F. Ann

Sol:

$$\begin{aligned} P.V. &= \frac{A}{i} (1+i) \\ &= \frac{10,000}{0.08} (1+0.08) \\ &= 1,35,000 \end{aligned}$$

Growing Perpetuity

When regular payment is received with growth for infinite period

Regular

$$P.V. = \frac{R}{i - g}$$

Immediate

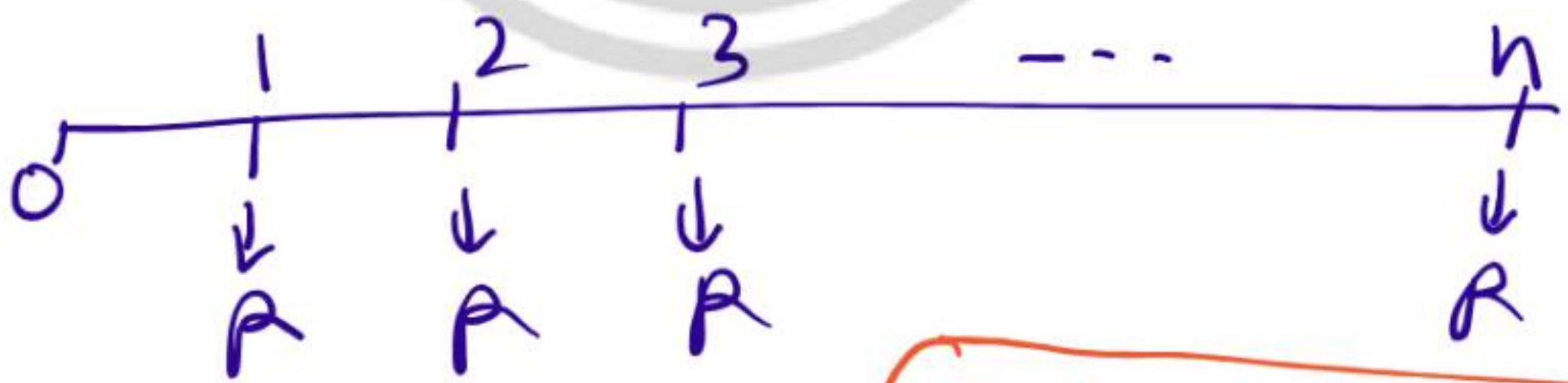
$$P.V. = \frac{A(1+i)}{i - g}$$

#

Sinking Fund

Saving money out of Profits
so that you can purchase an
Asset in future or you can
payoff a liability in future

Saved amount is invested in which
interest is earned

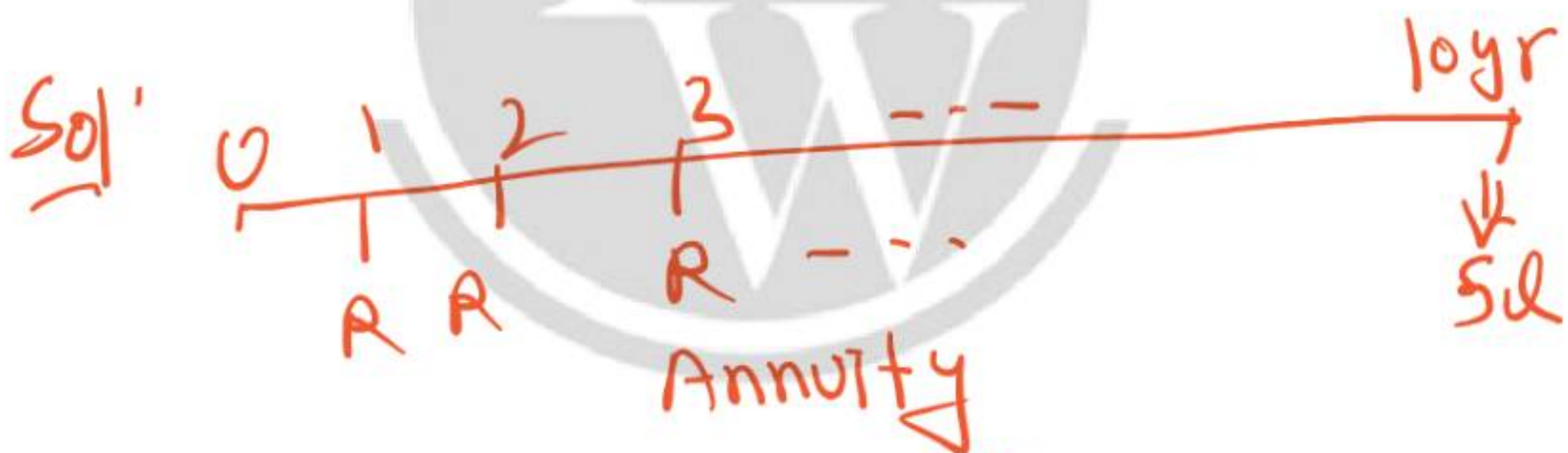


$R = \text{Saving}$

FoV. of Regular Annuity will be used

In this case benefit is received
in future

Q Suppose you have to pay off a liability after 10 years for that you require ₹ 500000 after 10 years. How much you should invest in a sinking fund which gives 12% P.A. compounded annually so that you can fulfill your future needs?



$$F.V. = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$500000 = A \left[\frac{(1.12)^{10} - 1}{0.12} \right]$$

$$500000 = A (17.54)$$

$$R = 28506$$

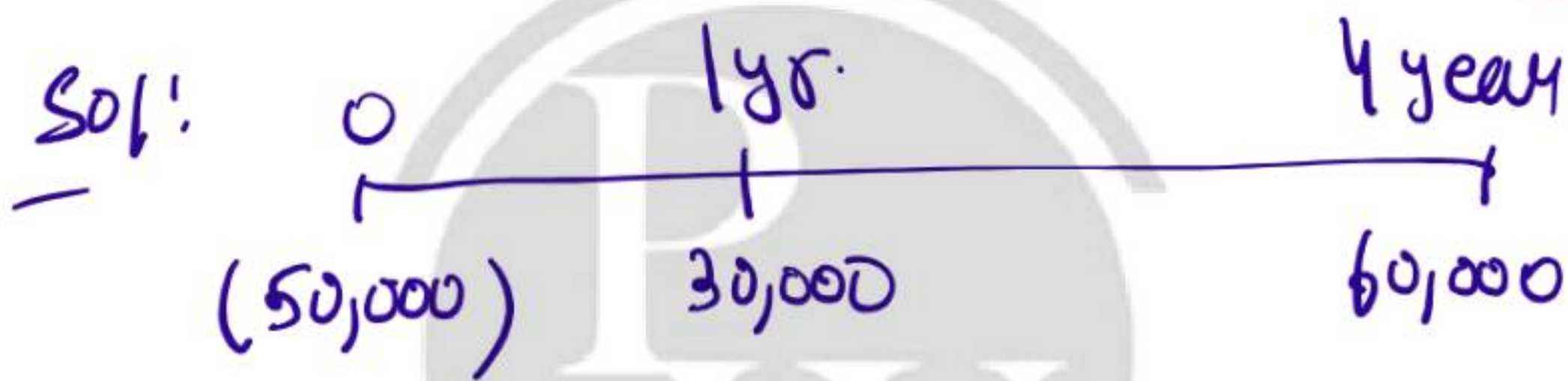
Net Present value

used for taking investment decisions

$$\text{NPV} = \text{Present value of all cash inflow} - \text{Present value of all cash outflow}$$

$$\left[\begin{array}{ll} \text{If } \text{NPV} \geq 0 & \text{Accept Proposal} \\ \text{If } \text{NPV} < 0 & \text{Reject Proposal} \end{array} \right.$$

q In a project £50,000 spend today.
 It generated £30,000 after one year
 & 60,000 after 4 years.
 Find NPV of project if
 discount rate is 10% Annually.



$$\text{NPV} = \frac{30000}{(1.10)^1} + \frac{60,000}{(1.10)^2} - 50,000$$

$$= 27272.72 + 49586.77 - 50,000$$

$$= 26859.49$$

#

Leasing



Taking Asset on Rent
(For Long Period)

owner of Asset



Lessor
मालिक

user of Asset



Lessee
किरायेदार

In this type of question
Purchase value of Asset
will be given.

But problem is we don't

know how much should be
the Annual Rent?



use P.V. of Regular Annuity
to find Reasonable Rent



$$\text{Cost of Asset} = \text{Rent} \left\{ \frac{1 - (1+i)^{-n}}{i} \right\}$$

eg. Mr. X can purchase a
machine for £50,000 Today or
he can take the machine on lease
for 7 years in £9000 Annual
Rent. which option is better?

Purchasing or Leasing of money is worth 10% Annually.

Sol: In this question find reasonable rent & then compare it with actual Rent (i.e. 9000)

$$50,000 = \text{Rent} \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$50,000 = R \left[\frac{1 - (1.10)^{-7}}{0.10} \right]$$

$$50,000 = R (4.8684)$$

$$R = 10,270$$

Reasonable Rent is £ 10,270

But he is getting the machine on lease in much more cheaper price (i.e. 9000)

So He should go for leasing

Valuation of Bond

Price at
which Bond
should be
Purchased

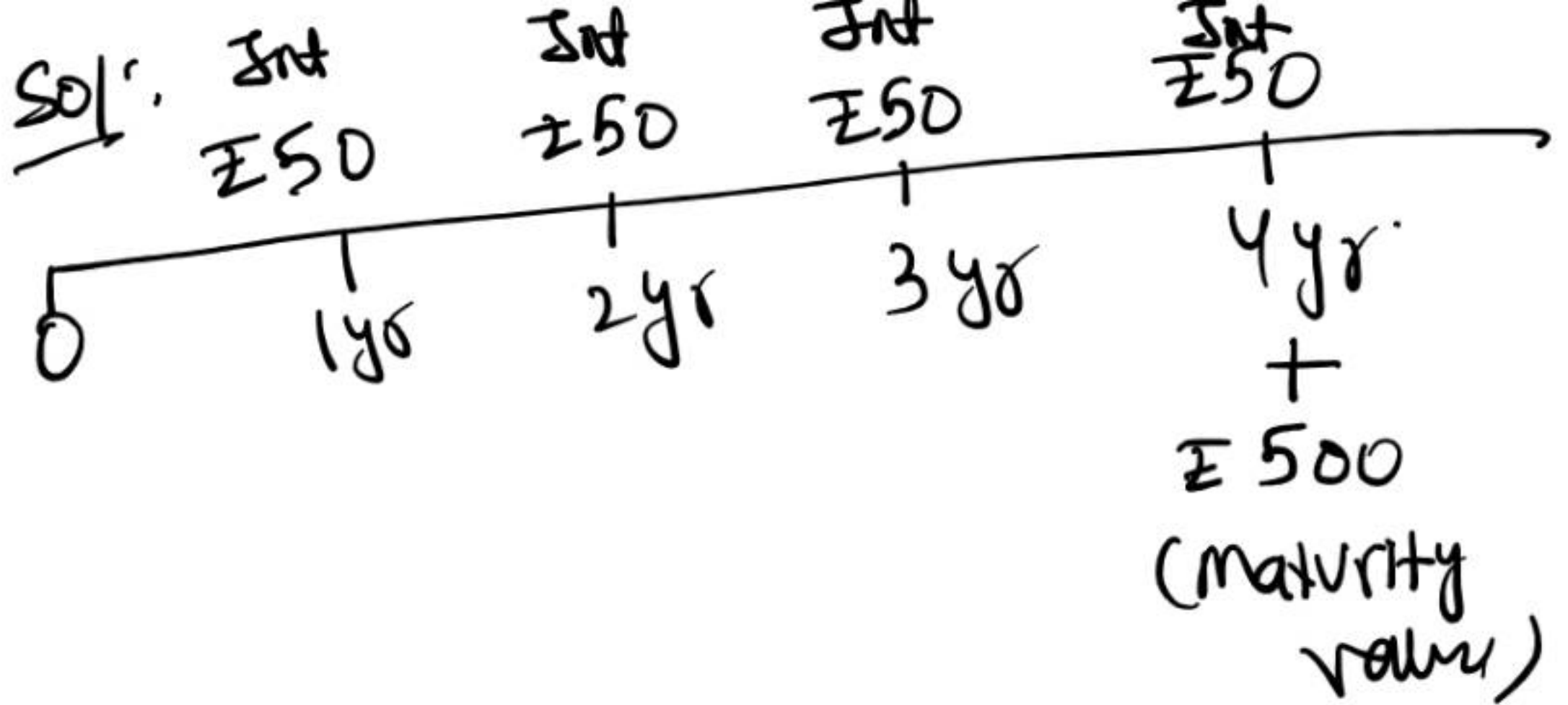
=

Present
value of
All future
interest

+

Present value
of Bond's
maturity
value

Q Bond of ₹ 500 at which
interest rate is 10% & maturity
period is 4 year. At what price
it should be purchased if investor
wants 15% return.



Price of Bond

$$= \frac{50}{(1.15)^1} + \frac{50}{(1.15)^2} + \frac{50}{(1.15)^3} + \frac{50}{(1.15)^4} + \frac{500}{(1.15)^4}$$

$$= 43.47 + 37.80 + 32.87 + 28.58 + 285.87$$

$$= \text{£} 428.59$$

C. A. U. R.

Compound Annual Growth Rate

Time	2010	2011	2012	2013
Revenue	100	110	140	160

find CAUGR

Sol:

$$P = 100$$

$$A = 160$$

$$\text{Time} = 2013 - 2010 = 3 \text{ years}$$

$$A = P(1+r)^t$$

$$160 = 100 [1+r]^3$$

$$\frac{160}{100} = (1+r)^3$$

$$\left(\frac{160}{100}\right)^{1/3} = 1+r$$

$$r = \left(\frac{160}{100}\right)^{1/3} - 1$$

OR

USE Direct formula

$$\begin{aligned} \text{CAGR} &= \left[\frac{V(t_n)}{V(t_0)} \right]^{\frac{1}{t_n - t_0}} - 1 \\ &= \left(\frac{160}{100} \right)^{\frac{1}{2013 - 2010}} - 1 \end{aligned}$$

$$= \left(\frac{160}{100} \right)^3 - 1$$

$$= 1.1696 - 1$$

$$= 0.1696$$

or
16.96%

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